TURBULENCE AND HEAT TRANSFER CHARACTERISTICS OF LOW REYNOLDS NUMBER VISCOELASTIC FLUID FLOW IN A SERPENTINE CHANNEL

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Abstract

This paper describes the study on the flow pattern and heat transfer characteristic of viscoelastic fluid in a serpentine channel under low Reynolds number condition. The Reynolds number was varied in the range of 0.1< Re < 5 that was found in this study to cover the regime from steady laminar flow to unsteady elastic turbulent flow. Average heat transfer coefficients of the wall were measured under isothermal wall condition. And pressure loss and flow visualization measurements were also carried out. Two types of fluids were used as working fluids: one was 64.4wt% sucrose solution that was a Newtonian fluid, and the other was a 64.4wt% sucrose solution with which 500ppm polyacrylamide was mixed that was a viscoelastic fluid. In the visualization measurement, strong flow fluctuations were observed in the serpentine channel as Re increased in the case of the viscoelastic fluid, while the flow remained steady in the case of sucrose solution. An effective heat transfer enhancement compared with the sucrose solution case was observed as Re increased due to this flow fluctuation. An increase of the pressure loss was accompanied, too. However, a reasonable enhancement was obtained considering the total performances.

INTRODUCTION

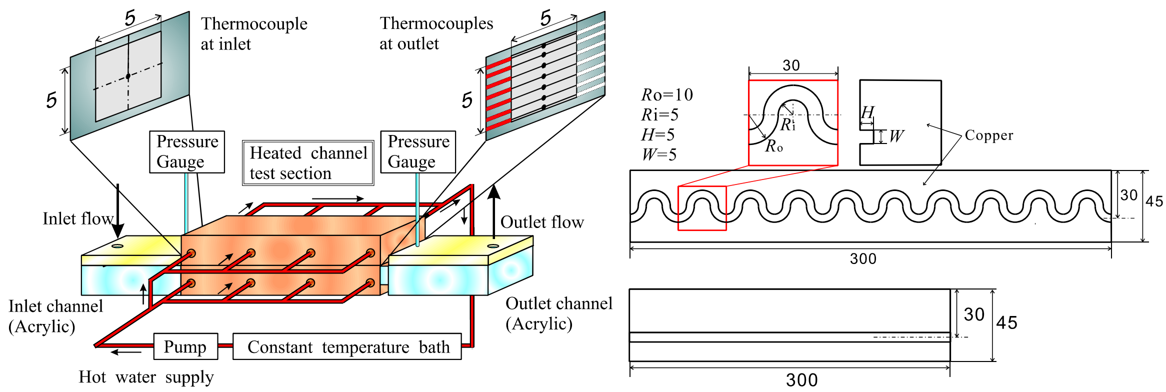
Enhancement technologies for convection heat transfer of laminar flows in channels have been keenly studied in the past half century. Various techniques have been developed in result, as typified by those such as using fins and corrugated channels to increase the heat transfer area and generating secondary flows, or applying jet flows, heat pipes, porous media and microchannels.

An alternative solution to the above geometrical and mechanical improvements of the channel is changing the properties of the operating fluid. The so-called "Nano-fluids" is a fluid in which nano-meter size particles are mixed in water or other solvents (Das et al., 2007). Although the physical reason is not fully understood, it is known that the heat transfer coefficient in channels can be increased compared with water in this case. Another working fluid that is of significant interest is viscoelastic fluids, for example polyacrylamide water solutions. Such fluid possesses not only viscous property but also elastic property. Providing high shear stress to the fluid by the shear flow will cause the polymers solved in the fluid to contract, expand and rotate leading to an appearance of the elasticity that produces additional normal stresses in the flow. Further, in combination with a certain flow structure, the elasticity of the fluid and the accompanied normal stress will increase the flow instability. The increase of the flow instability evokes unsteady flow with large fluctuation in the channel even in very low Reynolds number regime. Such unsteady flow is believed to possess a high potential to enhance the fluid mixing and wall heat transfer.

The influences of the rheological properties of the viscoelastic fluid on laminar heat transfer have been keenly studied in the past several decades, particularly in the case of straight channel. Hartnett et al. (1985) and Chunbo & Hartnet (1992) have reported from their experiment that the heat transfer coefficient in the case of viscoelastic fluid flow was higher than the Newtonian fluid flow case. The main cause was believed to be attributed to the secondary flow which was generated and enhanced by the normal force difference. Naccache et al. (1996) conducted a numerical simulation on the non-Newtonian laminar heat transfer in a rectangular duct and reported that the heat transfer was strongly enhanced due to the secondary flow. Norouzi et al. (2009) have numerically investigated the heat transfer characteristics of viscoelastic flow in a curved duct. The results showed that the first normal stress differences encourage the secondary flow generation and enhance the heat transfer while the second normal stress differences suppress heat transfer efficiency.

Flow instability of viscoelastic fluid flow has also been studied with strong interest. Larson et al. (1990) studied the flow instability of Taylor-Cuette flows in rotating coaxial cylinders. Joo & Shaqfeh (1994) investigated the mechanism of the increase in the flow instability of Taylor-Dean flows in coaxial cylinder.

Recently, several studies on the mixing characteristics of viscoelastic fluid in a serpentine-curved channel under low Reynolds number condition have been published in the literatures. Groisman & Steinberg (2001) carried out flow visualization and point measurement of the flow velocities using laser Doppler anemometer. Flow fluctuation was observed even in the order of $Re \sim 10$, accompanying the enhancement of the fluid mixing. Teodor et al (2004) and Feng et al. (2010) conducted a visualization experiment in serpentine microchannels using fluorescent dye. Even in this scale, production of the unsteady flow and enhancement in the fluid mixing was observed. Tamano et al. (2009) made PIV measurements in curved channels with circular cross-section and found a pair of vortices similar to the Taylor vortices in the time-mean velocity fields. All these results indicate the possibility of effectively enhancing the mixing performance or moreover mass and heat transfer at the channel wall. To the knowledge of the authors, however, there are no experiments conducted to link such unsteady flow and the fluid mixing with the heat/mass transfer performance. It is essential to understand the heat transfer characteristic of viscoelastic fluid flows in serpentine channels in order to provide an insight for the feasibility of using the viscoelastic fluids as working fluids, and to aid the designing of an effective channel shape and conditions in order to apply this technique to the fields of thermal engineering, chemical engineering and medicine.



(a) Apparatus *(b) Channel dimensions*

Figure 1: *Experimental apparatus used for the heat transfer measurement.*

From this background, measurement on heat transfer coefficient of viscoelastic fluid flow in a serpentine channel with a square cross-section is carried out under isothermal wall condition in this study. The pressure loss in the channel is measured together in order to evaluate the relationship between the heat transfer enhancement and pressure loss penalty. Further, visualization experiments are carried out to measure the flow behavior and discuss the mixing characteristic in the channel. Two types of working fluids are investigated. One is a viscoelastic fluid that is a sucrose water solution with which polyacrylamide of 500ppm is mixed. And the other is a sucrose water solution that is a Newtonian fluid used in order to make comparison with the viscoelastic fluid case.

EXPERIMETNAL METHOD

Experimental apparatus

Figure 1 shows the schematic diagram of the experimental apparatus applied to the heat transfer measurement. The channel consists of the three parts: the inlet and outlet channel parts, and the test section, the channel walls of which are heated under isothermal condition. The working fluid was supplied to the channel by a pump driven by pressurized air. The mass flow rate was measured by measuring the mass of the fluid flowing out from the outlet of the channel using an electronic weighing machine. The average values was used to calculate the flow rate, the accuracy of which was 0.6ml/min.

For the channel, the inlet and outlet units were made of acrylic, to which a straight channel was embedded. The test section was made of copper in order to produce an isothermal condition to the wall. As shown in Fig. 1 (a), cylindrical channels were transversely drilled to the wall to which constant temperature water was supplied from the thermostat bath to keep the wall temperature of the channel constant.

Two types of channel, the schematic and dimensions of which are shown in Fig. 1 (b), were investigated in this study. Both channels had a square cross-section of 5mm in width and height. The serpentine channel had a periodical shape, which consisted of 10 repeating units of circular curved parts. The inner and outer radii were 5 and 10mm, respectively. The length of the straight channel was L = 0.2m, while the length of the serpentine channel measured along the curvature was L = 0.471m.

***Table 1**: Flow conditions in the experiment.*

Fluid	Channel shape	Q [ml/min]	Re	De	Wi
Sucrose	Serpentine	25.3 – 138	1.01 – 4.29	0.59 – 2.48	–
PAAm	Serpentine	20.5 – 148	0.33 – 1.92	0.19 – 1.11	7.0 – 50
PAAm	Straight	24.6 – 70.0	0.28 – 0.88	–	8.0 – 24

The inlet and outlet parts were attached to the test section and sealed with silicon sealant. 5mm holes were drilled through the top plate at the end of the channels. Tubes were connected to these holes supplying fluids to the channel from the pump, and leading the outlet fluid to a container to measure the mass flow rate.

Heat transfer measurement

In order to measure the fluid temperature for bulk mean temperature calculations, temperature measurement units were placed at the inlet and outlet of the cupper test section. The fluid temperature at the inlet, T_i, was measured by a sheath K-type thermocouple of 0.5mm in diameter having its end point positioned at the center of the channel cross section. Temperature at a single point only was measured since a uniform temperature distribution was expected for the inlet flow. The fluid temperature at the outlet T_o was measured by 7 K-type thermocouples located evenly in the height direction in the middle of the channel width. Since the available temperature distributions obtained from this measurement are only in one-dimensional form, expansion of the temperature distribution from one- to two-dimensional directions is necessary. In this study, the fluid temperature was assumed to have a similar distribution to the measured one at z = 0 in the y and z directions. The temperature of the heated cupper part was measured by K-type thermocouples, which were embedded in the cupper surface. The variation of the temperature was 0.5 °C affirming that a constant wall temperature condition was obtained at the channel.

Streamwise velocity distributions of a square cross-section in a laminar flow case (Shah, 1978) were used in the sucrose solution case. In the PAAm solution case also, we have assumed that the velocity distribution is similar to the laminar flow of Newtonian fluid case based on the discussion on the friction factor, which will be shown in the next section. The temperatures and velocities obtained from these processes were employed to calculate the bulk mean temperatures.

Pressure loss measurement

To measure the pressure loss in the test section, electronic pressure gauges (Keyence Co., AP-C30), the measurement resolution of which was 0.1kPa, were connected to the inlet and outlet of the test section through a 1mm hole drilled at the top wall. The static pressure at each position was measured and used to calculate the fanning friction factor, f , defined as Eq. (1) .

$$f = \frac{2\Delta P}{\rho {U_m}^2} \frac{D_h}{4L} \qquad (1)$$

D_h is the hydraulic diameter of the channel. U_m is the mean velocity.

Flow Visualization

Flow visualization using dye is carried out in this study to understand the flow behavior in the channel. Syringe pump (Nihon Konden Co., CFV-3200) was used to supply the ink though a

4mm inner diameter tube to a syringe needle, which was inserted to the channel through the top wall. The images of the visualization was recorded using a digital video camera (Sony, HDR-CX370V) measured from above the channel. The configurations of the test section are nearly the same as those of the heat transfer experiment, except that the top wall was replaced by transparent acrylic plate. Therefore, the sidewalls and bottom walls only were heated in the visualization measurement.

Working fluid preparation

Two kinds of working fluids were prepared for the experiment. One was a sucrose water solution (sucrose 64.4wt%), which was a Newtonian fluid. The other was the polyacrylamide (PAAm) water solution (sucrose 64,4wt%, NaCl 1wt%, PAAm 500ppm), which was a viscoelastic fluid. The fluids were prepared mixing the solute to water using a pot rotating mill. NaCl was mixed in the fluid to keep the polymer structure and fluid properties stable.

The fluid viscosity μ, density ρ and relaxation time γ were measured prior to the experiment. μ measured by the rheometer were $\mu = 1.0 \times 10^5$ and 1.0×10^5 for the sucrose and PAAm solutions, respectively. The relaxation time was 2~3s which was calculated by appying the Powell-Eyring (Hartnetl et al., 1985) model to the measured results using the rheometer. The heat capacity C_p and thermal conductivity λ, were derived from the empirical equation shown in the references of Gucker and Ayers (1937) and Werner et al. (2007) for the fluid temperature of 25°C.

Experimental conditions

The flow rate was change in the experiments, in order to examine the influence of the Reynolds number, *Re*, on the heat transfer coefficient, pressure loss and flow behavior in the channel. The conditions are summarized in Table 1. *De* shown in the table is the Dean number, which is defined as Eq. (2).

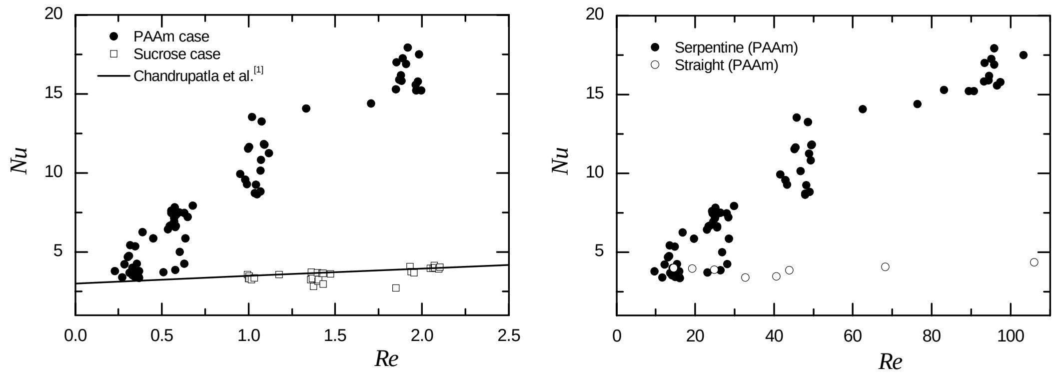
$$De = \frac{\rho U_m D_h}{\mu}\left(\frac{2W}{(R_i + R_o)}\right)^{0.5} \qquad (2)$$

W is the channel width, and R_i and R_o are the inner and outer radii of the curved channel. Dean number can be used to assess the magnitude of the curvature effect on the flow structure, for example the generation of the Taylor-Dean vortices. *Wi* is the Weissenberg number and is defined as follows.

$$Wi = \gamma\left(\frac{U_m}{D_h}\right) \qquad (3)$$

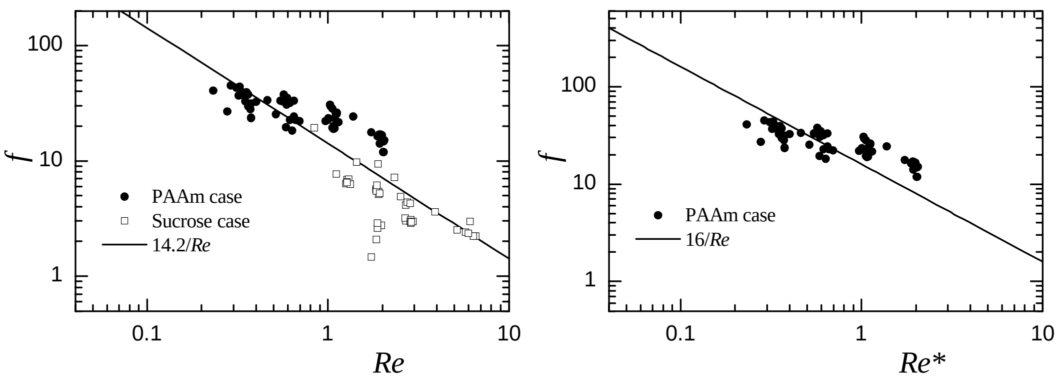
RESULTS AND DISCUSSION

Figure 2 shows the relationship between the average Nusselt number, Nu_m, and the Reynolds number, *Re*, for the sucrose and PAAm solutions. The analytical results for the Nusselt number of laminar flow in a straight channel with square cross-section is depicted in the figure for comparison. This analysis also considering the entrance region effect, and is derived by Chandrupatla & Sastri (1977).



(a) Sucrose vs PAAm solutions *(b) Serpentine vs straight*

Figure 2: *Average Nusselt number distributions: (a) comparison of the sucrose and PAAm solutions in serpentine channel, and (b) comparison of the serpentine and straight channels in the PAAm solution case.*



(a) Re vs f *(b) Re* vs f*

Figure 3: *Friction factor distributions: (a) relationship with Reynolds number, and (b) relationship with modified Reynolds number based on the power law.*

The Nusselt number Nu_{m} of the sucrose solution case slightly increases with *Re* due to the development of the thermal boundary layer at the inlet. Further, the result agrees well with the analytical one showing the validation of the measurement. Therefore, the flow in the serpentine channel is flowing along the curved channel without accompanying any secondary flows, which is consistent with the small value of the Dean number shown in Table 1.

In the PAAm solution case, Nu_{m} increases markedly as *Re* increases, and shows a considerable heat transfer enhancement compared to the sucrose solution case. Nu_{m} appears to approach to the value of sucrose solution case as *Re* decreases. Therefore, it is expected that the increase of Nu_{m} is related more to the production of secondary flow than the change in the velocity distribution due to the non-Newtonian behavior such as shear thinning.

In order to see whether the heat transfer enhancement is achieved by the viscoelasticity of the fluid only, or if it is a combinational effect of the viscoelastic fluid and the curvature of the serpentine channel, heat transfer measurement was carried out for a straight channel using the same PAAm solution. The results are shown in Fig. 2 (b). Since the channel length is different between the two cases, Graetz number *Gz*, defined by Eq. (4), is taken as the abscissas axis in this case.

$$Gz = \mathrm{Re}\frac{D_h}{L}\frac{\rho\mu}{\lambda} \tag{4}$$

In the figure, we can see that the Nu_m does not increase in the straight channel case for the *Re* conditions investigated in this study as it did in the serpentine channel case. It remains nearly constant at a value close to $Nu_m \sim 3$, which is the value of fully developed flow and thermal boundary layers in a square duct. This shows that the increase of heat transfer coefficient can be obtained only by the combination of the viscoelastic fluid flow and the serpentine channel, at least under the conditions of the Reynolds number considered in this study.

Figure 3 shows the relationship between the fanning friction factor, *f*, and the *Re* in both solution. The solid line in the figure presents the value of 14.2/*Re*, which shows the friction factor of a Newtonian fluid flow in a square duct. Although the results in the sucrose solution case shows a relatively smaller value compared with the solid line, the scattering plots show a reasonable agreement with the theoretical value. Since the Reynolds number and Dean number are both very small, it is believed that no noticeable secondary flow is generated in the channel. Therefore, same as in the case of Nu_m shown in Fig. 2, the results indicate that the flow in the serpentine channel in the sucrose solution case can be considered to be a laminar flow in a straight duct with square cross-section.

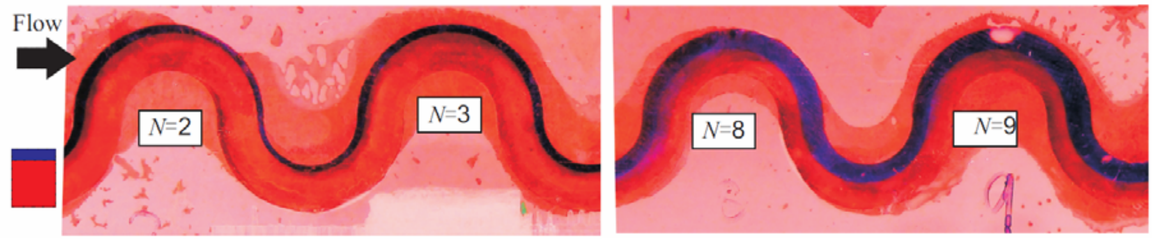
On the other hand, *f* in the PAAm solution case shows a different tedemcy. In the lower *Re* area, *f* shows a slightly smaller value than the solid line similar to the sucrose solution case. However, a large increase of *f* is observed in the region of larger *Re* , in which *f* shows a greater value than 14.2/*Re*. The reason for this increase can be two-fold. One is the change in the velocity distribution due to the viscoelastic properties of the fluid. In this case, the velocity gradient increases in the area adjacent to the channel wall, and a more flat distribution is obtained at the channel center leading to a larger friction at the channel wall. The other reason is the generation of the flow fluctuation or secondary flow in the channel.

In order to clarify each influence in the friction factor shown in Fig. 3 (a), the relationship between *f* and *Re* is replotted by using the modified Reynolds number, *Re**. *Re** is calculated by Eq. (5).

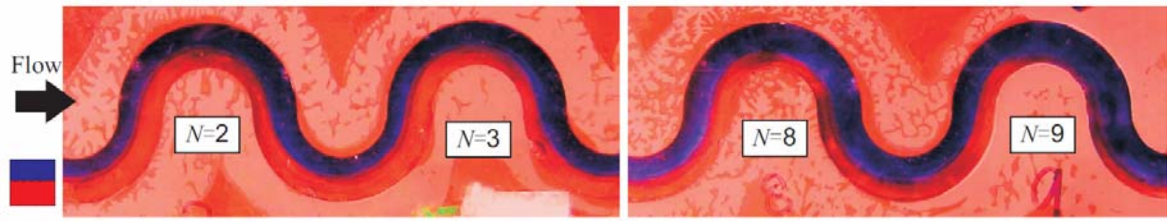
$$\mathrm{Re}^* = \frac{\rho U_m{}^{2-n} D_h{}^n}{8^{n-1} K (b + a/n)^n} \tag{5}$$

This equation considers the influence of the viscoelasticity of the fluid on the velocity profile, and shows the relationship between the friction factor and the modified Reynolds number. In this case, the relationship between *f* and *Re** can be shown as *f* = 16/*Re** for viscoelastic fluid flows. The parameters *a* and *b* appearing in Eq. (5) are used to describe the effect of the cross-sectional shape of the channel. As a square cross-section, a = 0.21 and b = 0.68 are employed in this study. *K* and *n* are calculated from the logarithm law described in the references of Hartnetl et al. (1985) and Kozicki et al. (1966). The results are shown in Fig. 3 (b).

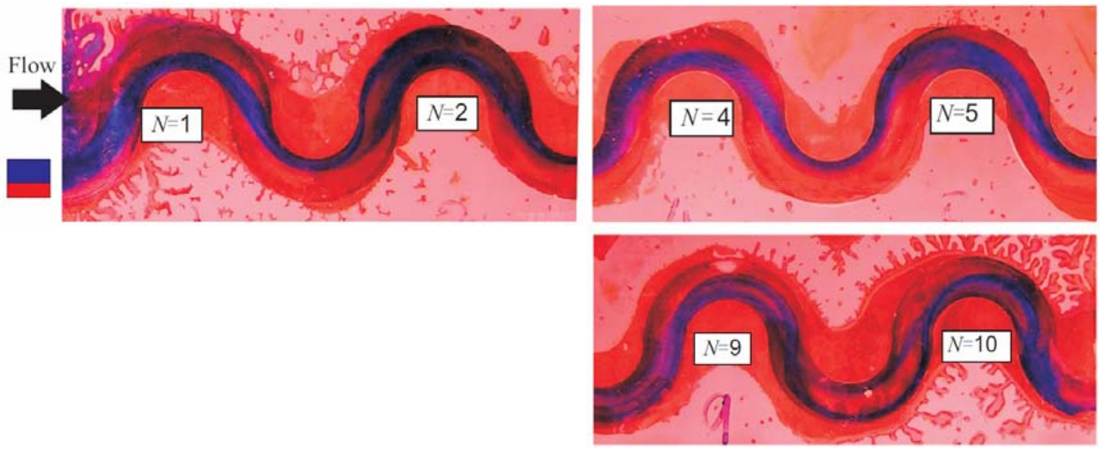
In Fig. 3 (b), the present results decreases and separatures further from the solid line (theoretical value) in the regions of small *Re**. Then, *f* increases, crosses the line, and deviates from the solid line in the positive direction as *Re** increases. Therefore, under the small *Re** conditions, the flow is believed to be steady and also having a velocity profile



(b) Sucrose solution case (Re = 2.49, De = 1.44)



(b) PAAm solution case (Re = 0.369, De = 0.213)



(c) PAAm solution case (Re = 1.64, De = 0.945)

Figure 4: *Flow visualization using dye in the cases of (a) sucrose solution at steady state, (b) PAAm solution at steady state, and (c) PAAm solution at unsteady state.*

similar to a Newtonian fully developed laminar flow in a square duct. On the other hand, as Re^* increases, the flow becomes unsteady. And, thus, additional increase of the friction factor is obtained due to the generations of the flow fluctuation and secondary flow. It should be marked that the transition region in which the Nu_m and f start to increase matches with each other reasonably well showing a change in the flow structure is occurring.

Finally, the visualization results are shown in Fig. 4. The figures show the snapshots taken at several streamwise positions in the cases of sucrose and PAAm solutions. In the PAAm solution case, the results of Re = 0.369 and 1.65, lower and higher Re conditions, are shown to discuss the influence of the Re on the flow characteristics.

In Fig. 4 (a), the streamlines of the ink flows along the curvature of the serpentine channel. This pattern remained steady throughout the movie showing that a steady flow without any noticeable secondary flow is formed in the channel. In Fig. 4 (b) showing the PAAm solution case of Re = 0.369, a similar pattern is observed. Although some disturbance in the streamlines of the dye was observed instantaneously in other periods of the movie, the flow could be considered to be in steady state.

On the other hand, in Fig. 4 (c), one can see that the ink patterns are greatly disturbed in all streamwise positions, showing that unsteady flow with flow fluctuation is produced in the channel. In the movie, the fluctuation of the flow shedding downstream of the channel was clearly observed. It is clear now that this flow fluctuation causes the heat transfer enhancement and increase of the pressure loss in the serpentine channel shown in the previous figures. It is difficult to estimate the exact flow pattern from the visualization results since the flow appeared to have a three-dimensional structure and the camera time resultion was not high enough to follow the unsteady flow behaviour in detail. We believe that, however, in the early stages of the fluid viscoelasticity influencing the flow, a deviation of the spanwise velocity distribution is produced at the curved part of the channel due to the radial flow heading outwards. This generation of radial flow should be based on a similar reason described in the instability studies of Joo and Shaqfeh (1994). That is, the hoop-stress produced by the polymer motion increases the flow instability leading the generate the Taylor-Dean voctices or a similar secondary flow. Once such asymmetric flow or vortices are generated, then the flow fluctuation could be enhanced at the inflection point of the serpentince channel due to the change of the flow directions.

CONCLUSION

Heat transfer and pressure loss measurements, and flow visualization were carried out for viscoelastic fluid flow in serpentine channels. The conclusions obtained from the experiments are summarized as follows:

1. Enhancement of heat transfer performance was observed in the PAAm solution (viscoelastic fluid) case compared with the sucrose solution (Newtonian fluid) case. In lower Reynolds number region (*Re* ~ 0.5), the average Nusselt number in the PAAM case showed a similar value with the sucrose case, and then increased markedly as the *Re* increased implying that a transition of the flow from steady to unsteady state occurred. Further, average Nusselt number Nu_m in the case of the PAAm solution in a straight channel did not show a large difference with the sucrose case indicating that the enhancement effect could be obtained only by the combination of the channel curvature and the fluid viscoelasticity.

2. Pressure loss increased markedly as the *Re* increased. By discussing the relationship between *Re* and the friction factor *f*, using the modified *Re* based on the power law fluid, showed that the increase of the pressure loss is mainly attributed to the flow fluctuation and not to the power law velocity distributions.

3. The tendency of the flow behavior shown by the visualization results corresponded well with heat transfer and pressure loss measurements mentioned above. A large scale vortex shedding and a deviation of the streamline pattern were observed, which appeared to have a strong three-dimensional structure.

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